Nonlinear state-dependent delay modeling and stability analysis of Internet congestion control

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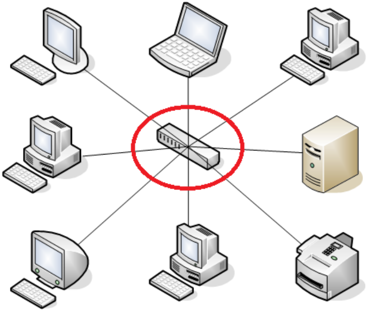
The congestion problem in networks

Network Elements

- Buffers/Servers/Routers
- Media
- Users

Congestion problem - QoS deterioration

- Data loss
- Too large delay



Need for protocols for congestion control

Problem definition

Modeling problem

- Precisely represent each element (user, medium, buffer)
- Provide precise adapters/interfaces

Modular construction of networks models

Congestion control problem

- Define efficient protocols achieving performance specifications
  - Fairness
  - Efficiency
  - Cross-traffic adaptation
- Analyze network stability and dynamic performance
  - Local vs. global stability
  - Static and dynamic performance
  - Delays effects on stability

How to model networks ?

Packet level

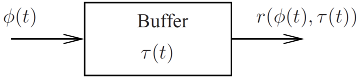
- Asynchronous discrete-time large-scale (hybrid) system
- Perfect for simulation (NS-2)
- Few tools available for analysis

Flow level

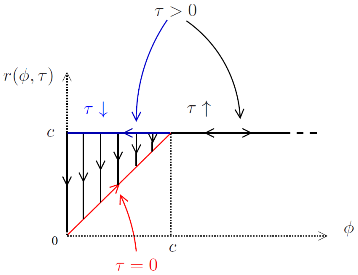
- Valid when packets size small w.r.t. transfer speed (e.g. kbit vs Mbit/s)
- Fluid-flow models, continuous-time
- Difficult to transpose packets level effects to a flow level
- Good models for representation and analysis

Buffer model (infinite capacity)

$$
\begin{aligned}
\dot{\tau}(t) &= \frac{1}{c}\left[\phi(t) - r(\phi(t),\tau(t))\right] \\
r(\phi(t),\tau(t)) &= \begin{cases} c & \text{if } \mathcal{C}(t) \\ \phi(t) & \text{otherwise} \end{cases} \\
\mathcal{C}(t) &= [\phi(t) > c] \text{ or } [\tau(t) > 0] \\
\dot{\tau}(t) &\in [-1, +\infty)
\end{aligned}
$$



- Hybrid linear model with linear constraints
- Flow integrator
- Aggregated flows
- Queue saturation $\rightarrow$ 3rd mode



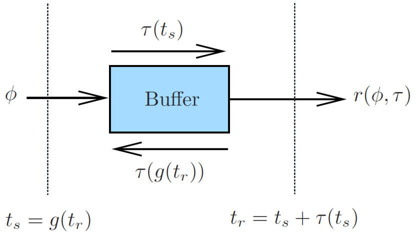
Queuing delay map

The time is not universal

- Temporal order of reaction
- Different models for different reference times

Sending time t_s as a reference

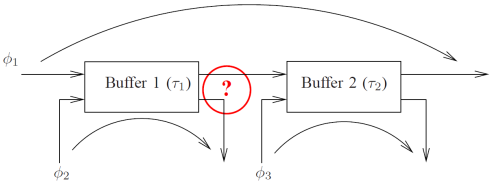
- Forward operator $f(t) = t + \tau(t)$.
- Reception time $t_r = f(t_s)$
- Intuitive, easy for modeling



Reception time t_r as a reference

- Backward operator $g = f^{-1}$ exists iff $\phi > 0$
- Sending time $t_s = g(t_r)$
- Less intuitive but better for analysis

Extended buffer model (1)



Problem of flow separation

- Crucial for interconnections description
- Flows are aggregated in the previous model
- How to split up r into a sum of atomic r_i (if possible) ?

Extended buffer model (2)

Output flows - Closed form expression

$$\dot{\tau}(t) = \frac{1}{c}\left[\sum_i \phi_i(t) - \sum_i r_i(\phi_t, \tau_t)\right]$$

$$r_i(\phi_t, \tau_t) = \begin{cases} \dfrac{\phi_i(g(t))c}{\sum_j \phi_j(g(t))} & \text{if } \mathcal{C}(g(t)) \\ \phi_i(t) & \text{otherwise} \end{cases}$$

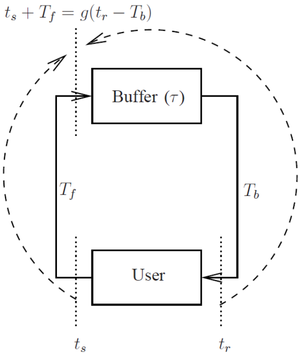
- $g(t) = t - \tau(g(t))$
- Delayed flow proportion
- Can be extended to more complex buffers, e.g. multiple output capacities

Forward vs. Backward protocol model

Forward protocol model - t_s is the reference

$$\begin{aligned}
\dot{z}(t_s + RTT_{t_s}) &= \mathcal{P}(z(t_s + RTT_{t_s}), \tau(t_s + T_f), T) \\
w(t_s + RTT_{t_s}) &= h(z(t_s + RTT_{t_s})) \\
RTT_{t_s} &= T_f + \tau(t_s + T_f) + T_b
\end{aligned}$$

- Not very easy to work with



Backward protocol model - t_r is the reference

$$\begin{aligned}
\dot{z}(t_r) &= \mathcal{P}(z(t_r), \tau(g(t_r - T_b)), T) \\
w(t_r) &= h(z(t_r)) \\
\tau(g(t_r - T_b)) &= \tau(t_r - T_b - \tau(g(t_r - T_b)))
\end{aligned}$$

- Standard form for dynamical systems
- Implicit state-dependent delay !

Complete network model

Single-buffer/Single-user

$$
\begin{aligned}
\dot{\tau}(t) &= \frac{1}{c}\left[\phi(t) - r(\phi(t), \tau(t))\right] \\
\dot{z}(t) &= \mathcal{P}(z(t), \tau(g(t-T_b)), T_f, T_b) \\
w(t) &= h(z(t)) \\
\phi(t) &= \Phi(w(t), \tau(t), T_b, T_f)
\end{aligned}
$$

- Functional Φ converts windows sizes into flows [Jacobsson]

$$
\phi(t) = \frac{w(t-T_f)}{T+\tau(t)}, \quad \phi(t) = \frac{w(t-T_f)}{T+\tau(t)} + \dot{w}(t-T_f)
$$

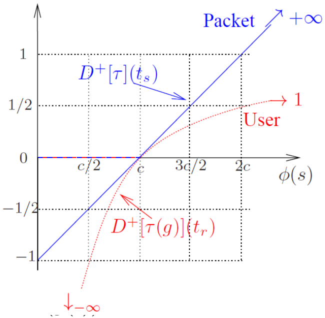
Delays properties

Propagation delays T_f, T_b

- Constant delays
- Bounded

Queuing delay $\tau(g(t - T_b))$ - Single buffer case

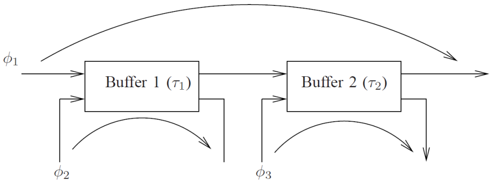
- Bounded in practice
- Derivative belongs to $(-\infty, 1)$:



$$D^+[\tau(g)](t) = \begin{cases} 1 - \dfrac{c}{\phi(g(t))} & \\ 0 & \text{otherwise} \end{cases}$$

- Well-posedness problems do not occur ($\dot{\tau}(t) < 1$)
- Many results on time-delay systems can be applied (Lyapunov-Krasovskii Theory)

Queuing delay $\tau(g(t-T_b))$ - Multiple buffer case



- Two delays: $\tau_1(g_1(g_2(t)))$ and $\tau_2(g_2(t))$
- $D^+[\tau_1(g_1(g_2))](t)$ given by

$$\begin{cases} \dfrac{(\phi_1(g_1(g_2)) + \phi_2(g_1(g_2)) - c_1)c_2}{(\phi_1(g_1(g_2) + \phi_2(g_1(g_2)))\phi_3(g_2) + \phi_1(g_1(g_2))c_1} & \text{if} \quad \mathcal{C}_1(g_1(g_2)) \text{ and } \mathcal{C}_2(g_2) \\ 1 - \dfrac{c_1}{\phi_1(g_1(g_2)) + \phi_2(g_1(g_2))} & \text{if} \quad \mathcal{C}_1(g_1(g_2)) \text{ and not } \mathcal{C}_2(g_2) \\ 0 & \text{otherwise} \end{cases}$$

- May exceed one when $c_2 > c_1$.

FAST-TCP

FAST-TCP protocol

$$\dot{w}(t) = \gamma\left(-\frac{\tau(g(t-T_b))}{T+\tau(g(t-T_b))}w(t)+\alpha\right)$$

- γ tunes the bandwidth of the protocol (control sense)
- α rules out the bandwidth-delay product (communication network sense)
- Network equilibrium point (buffer+user):

$$\tau^* = \frac{\alpha}{c(1-\delta^*)}, \quad w^* = \alpha(1+T/\tau^*), \quad \phi^* = \alpha/\tau^*$$

- Equilibrium flow independent of the propagation delay (also in the multiple users case)

Stability Analysis

The single-user/single-buffer network is

- globally exponentially stable without delays (Lyapunov theory)
- locally exponentially stable independently of the delay when

$$\tau^* > T \text{ where } \tau^* = \alpha/c \quad \text{(small gain)}$$

- locally delay-dependent exponentially stable when

$$\tau^* < T, \quad \tau^*(T-1) + T^2 \leq 0 \quad \text{(quasipolynomials)}$$

- locally delay-dependent exponentially stable when

$$\tau^* < T, \quad \tau^*(T-1) + T^2 > 0, \quad \gamma < \frac{1}{\tau^*(T-1) + T^2}$$
$$\text{(quasipolynomials)}$$

Conclusion and Future Works

Conclusion

- Accurate models for FIFO buffers
- theoretical delay modeling and analysis
- Local stability analysis based on time-delay systems theory

Future works

- More general buffer models (priorities, multiple output links/capacities)
- Nonlinear stability analysis
- Study of limit cycles in more complex topologies

Thank you for your attention